Preliminary study on Dreamlet based compressive sensing data recovery

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Summary

We present a data recovery scheme in Dreamlet (Drumbeat-Beamlet, which is a type of physical wavelet) domain based on the concept of compressive sensing. The data are randomly sampled with sparse samples missing 50% of the original data. Data recovery is done by a basis pursuit decomposition method based on l_1-norm optimization to search for the most efficient representation (least residual with minimum coefficients). Numerical tests show that Dreamlet representation is more efficient (with less coefficients) and the data recovery process is faster (with fewer iterations) than the curvelet method.

Introduction

Compressive sensing (Candes, Romberg and Tao, 2006; Donoho, 2006; Tsaig and Donoho, 2006) provides a new way of data acquisition design to achieve sparse acquisition without sacrifice of the data quality. Randomized sampling was used to break the Nyquist sampling law and reduce the acquisition cost by sparse acquisition. The aliasing artifacts produced by undersampling using standard acquisition are transformed into harmless background noise in randomized acquisition (see Herrmann, 2010). The keys to the success of compressive sensing are to have a highly sparse representation of the seismic data and to have an effective reconstruction method for the accurate data recovery. It is known that the recovery of a signal from severely undersampled data can be done by solving a related l_1-regularization problem (Candes, Romberg and Tao, 2006; Donoho, 2006; Van den berg and Friedlander, 2008). In this study, we will show that Dreamlet representation (Wu et al., 2008, 2010; Geng et al.,,2009) is an efficient and sparse alternative method for data recovery in compressive sensing with respect to other transforms such as curvelet transform. This is due to the fact that Dreamlet is a type of physical wavelet defined on the observation plane, automatically satisfying the wave equation.

Summary on the concept of Compressive sensing

Suppose that we want to recover the original data $f \in \mathbb{R}^N$ from the acquired data $y = \mathbf{A}f$, $y \in \mathbb{R}^n$ and $n \ll N$. $\mathbf{A}$ is a $n \times N$ matrix, which is called the restriction operator that picks the acquired samples from the original data. The data set y will be an undersampled data set, may be called data by "compressive sensing". When the original data set (fully sampled) f has a sparse representation $x \in \mathbb{C}^N$ in certain transform domain $\mathbf{S}$ with $f = \mathbf{S}^T x$, the relation becomes

$$y = \mathbf{B}x \text{, with } \mathbf{B} = \mathbf{A}\mathbf{S}^T . \tag{1}$$

where ^{T} stands for the inverse transform. To recover x , the solution $\tilde{x}$ of equation (1) can be obtained by solving a l_1 regularization problem (Candes, Romberg and Tao, 2006; Donoho, 2006; Tsaig and Donoho, 2006)

$$\min \|x\|_1 \text{ subject to } \|\mathbf{B}x - y\|_2 \le \xi , \tag{2}$$

and the original data can then be recovered by $\tilde{f} = \mathbf{S}^T \tilde{x}$. Solution $\tilde{x}$ is equivalent to x if equation (2) meets two conditions: (a) x is sufficiently sparse, (b) the matrix $\mathbf{B}$ obeys a uniform uncertainty principle (with unit-normed columns). That is to say that the undersampling artifacts $z = \mathbf{L}x = \mathbf{B}^T\mathbf{B} - \alpha\mathbf{I}$ are incoherent, where parameter α is a scaling factor so that $diag(\mathbf{L}) = 0$ and $\mathbf{I}$ is the identity matrix. Condition (b) also shows the importance that the artifacts introduced by subsampling the original data are not sparse in the transformed domain.

Effective representation of seismic data

To recover the seismic data, we have to find a transform $\mathbf{S}$ which can sparsely represent the seismic data. Several works have been done to use Curvelet transform together with Compressive sensing to recover the seismic data from a random undersampled data (Hennenfent and Herrmann, 2008; Herrmann et al., 2008; Herrmann, 2010). The property of Curvelet transform makes it a good representation of the wavefront in the data. In our work, we have introduced Dreamlet transform whose atom is in fact a type of physical wavelet defined on the observation plane. The Dreamlet atom is defined as the tensor product of *Beamlet* atom (space-wavenumber atom) and *Drumbeat* atom (time-frequency atom), which obey the causal relation or dispersion relation. In this study, we will use Gabor frame as the beamlet and drumbeat atoms:

$$d_{\bar{t}\bar{\omega}\bar{x}\bar{\xi}}(x,t) = g_{\bar{t}\bar{\omega}}(t) b_{\bar{x}\bar{\xi}}(x) \tag{3}$$

where

$$g_{\bar{t}\bar{\omega}}(t) = W(t - \bar{t}) e^{-i\bar{\omega}t} \tag{4}$$

is the time-frequency atom: "drumbeat" with $W(t)$ as a Gaussian window function,, and

$$b_{\bar{x}\bar{\xi}}(x) = B(x - \bar{x}) e^{i\bar{\xi}x} \tag{5}$$

is the space-wavenumber atom: "beamlet" with $B(x)$ as a Gaussian window function. We see that a dreamlet atom is

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a time-space windowed (t-x localized) local plane wave-packet:

$$d_{\bar{t}\bar{\omega}\bar{x}\bar{\xi}}(x,t) = W\left(t-\bar{t}\right)B(x-\bar{x})e^{-i(\bar{\omega}t - i\bar{\xi}x)} \qquad (6)$$

where $\bar{t}$, $\bar{x}$, $\bar{\omega}$ and $\bar{\xi}$ are the local time, location, frequency, wavenumber, respectively. The time-space-frequency-wavenumber atom defined by (6) satisfies the causal solution of wave equation. The construction of time-space atom in this way can efficiently represent the seismic data, resulting in high sparsification of seismic data in dreamlet domain. In contrast, noise or artifacts normally do not have this property.

Criterion of data recovery

We will use signal-to-reconstruction-error ratio (SNR) to show the sparse recovery quality

$$SNR = -20\log\frac{\left\| f-\tilde{f}\right\|_2}{\left\| f\right\|_2} \qquad (7)$$

where f is the data vector, $\tilde{f}$ is the reconstructed data, so f - $\tilde{f}$ is the reconstruction error, and $\|f\|_2 = \left(\sum_i |f(i)|^2\right)^{1/2}$ is the L_2 norm of a vector f.

First we apply this criterion to compare different decomposition schemes. The l_1-norm optimization scheme used in compressive sensing is a decomposition method by basis pursuit (Mallat, 1998; Chen et al., 2001; Van den berg and Friedlander, 2008). In the algorithm, the problem is solved as a constrained optimization problem,

$$\min\|x\|_1 \text{, subject to } \mathbf{S}^T x = f \qquad (8)$$

We will call the solution as *searched coefficients*, in comparison to the *analysis coefficients* obtained by directly applying the Gabor transform to the data and thresholding.
Using the SEG-EAGE poststack data the comparison is shown in Figure 1, where the red line denotes the SNR as a function of sparseness ratio $p = k/N$, k is the number of largest *searched* coefficients; while the blue line is the SNR calculated by k largest *analysis* coefficients obtained from the Gabor transform. It can be seen that with the same sparseness ratio, the SNR obtained from *searched* coefficients is always larger than the SNR obtained from *analysis* coefficients. The reason comes from the redundancy and nonuniqueness of the frame decomposition. It seems that basis pursuit may be able to reach the optimum decomposition.

Numerical Examples of data recovery using dreamlets

We know that both Curvelet atom and Dreamlet atom are localized atoms, so it is important that the maximum length of adjacent missing traces is less than the length of the atom in space domain. Hennenfent and Herrmann (2008) proposed a jittered undersmapling scheme to control the maximum length. In our study, we use random sampling, which ensures that the condition (b) can be satisfied, and keep the maximum sampling interval less than the atom's length in space domain.

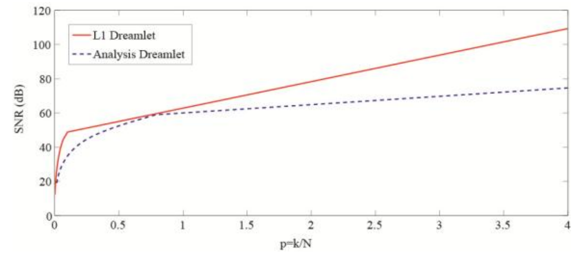

Figure 1: SNR for the nonlinear approximations of poststack data . The SNR is plotted as a function of sparseness ratio p=k/N, where k is the number of coefficients used to reconstruct data and N is the size of original data. The red line is from the decomposition by basis pursuit; the blue line is from that by Gabor transform.

Figure 2b shows a randomly subsampled synthetic shot gather with 50% of the traces missing in the receiver directions. Figure 2c shows the recovered data using Dreamlet transform with Gabor frames of redundancy of 2 in both time and space direction. Figure 2d shows the recovered data using Curvelet transform. Selected traces are plotted in Figure 3 to show the traces of the recovered data compared to the original data. It can be seen that both methods can recover the missing traces very well, and they can keep the existed traces remain correct. As shown in Figure 4b, random sampling introduces incoherent noise to the original amplitude spectral (Figure 4a) in the temporal frequency band, and after recovery, incoherent noises in the amplitude spectra of both Dreamlet (Figure 4c) and Curvelet methods (Figure 4d) are removed. However, as shown in Figure 5, Dreamlet method takes about 60 iterations to get the solution while Curvelet method takes around 400 iterations. The corresponding SNR are 20.99 dB for Curvelet and 25.88 dB for Dreamlet, respectively. Note also that the final number of dreamlet coefficients is smaller than that of the curvelets.

The second example is the poststack dataset of the SEG-EAGE model (Figure 6a). Figure 6b shows the randomly undersampled dataset with 50% of the missing traces. The recovered data obtain from Dreamlet method and Curvelet method are shown in Figure 6c and Figure 6d, respectively, and the corresponding amplitude spectra are shown in Figure 7. Figure 8 shows eight blocks of the coefficient matrix after Dreamlet decomposition. We can see that the random sampling behaves as incoherent noise in Dreamlet domain just the same as in Fourier spectra (Figure 7b). As shown in Figure 9, Dreamlet method takes around 60

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iterations to get the solution while Curvelet method takes around 950 iterations, and the corresponding SRR are 16.3 dB for Curvelet and 16.6 dB for Dreamlet, respectively.

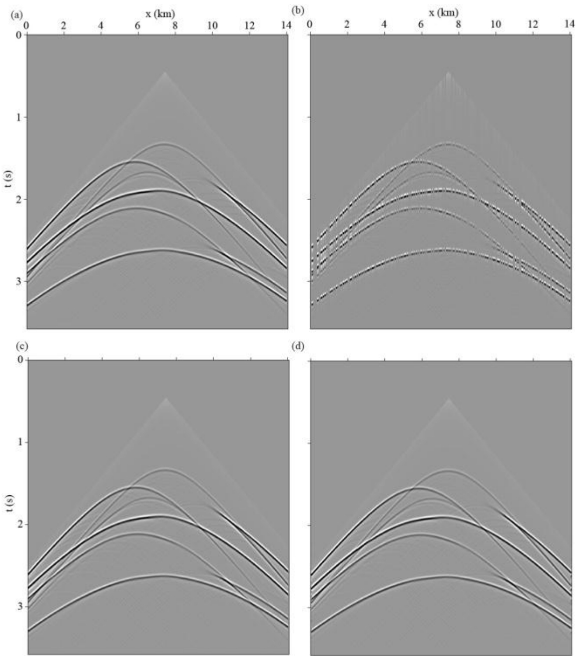

Figure 2: Randomly sampled data. (a) One shot gather for a 4 layer model, (b) shot gather with half traces missing, (c) Recovered data using Dreamlet transform, the Gabor frame has redundancy of 2 and window interval of 16 for both time and space direction, (d) Recovered data using Curvelet transform.

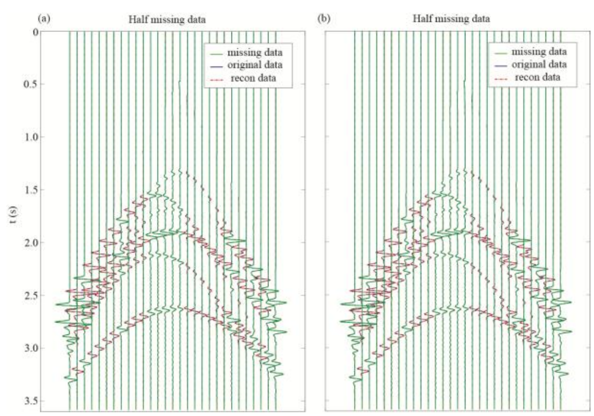

Figure 3: Comparison between traces of the original data and the recovered data in Figure 1, (a), using Dreamlet transform, (b), using Curvelet transform. In both figures, blue line stands for the original data, green line stands for the randomly undersampled data, and red line stands for the recovered data.

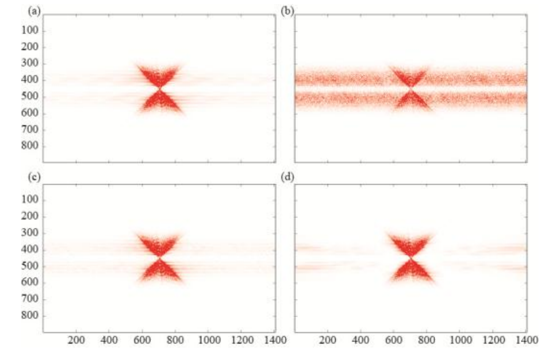

Figure 4: Fourier domain spectra of data in Figure 1. (a) Original data, (b) randomly sampled data, (c) recovered data using Dreamlet transform, (d) recovered data using Curvelet transform.

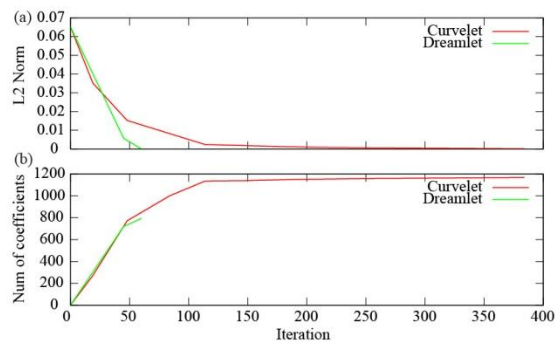

Figure 5: l_2-norm of the residual (top) and number of coefficients (bottom) as a function of iteration number for the data recovery of SEG Salt model poststack data set.

Conclusion

We study the use of Dreamlet for data recovery in seismic compressive sensing. We show that the coefficients calculated by solving l_1-norm optimization problem is sparser than the traditional Gabor decomposition, and we successfully recover the full data from the randomly sampled data. Dreamlet is a physical wavelet (wavelet atom satisfying wave equation) and the decomposition renders more efficient representation (less coefficients) with fast convergence (much less iterations) under the same data recovery quality (SNR), compared to Curvelet method. This study also shows the possibility to process the seismic data directly in the compressed domain after data recovery.

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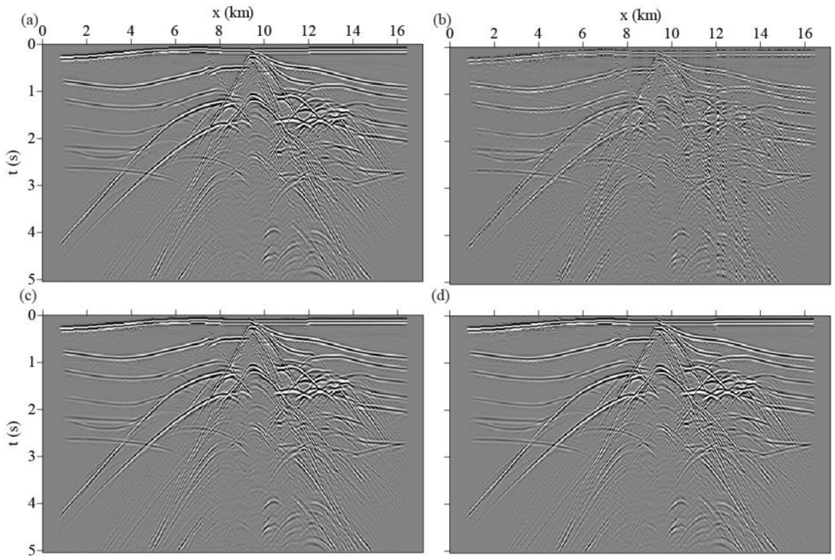

Figure 6: Data recovery of randomly missed data. (a) Original poststack dataset of SEG-EAGE model, (b) half traces are missed, (c) recovered data using Dreamlet transform. The Gabor frame has redundancy of 2 and window interval of 16 for both time and space directions, (d) recovered data using Curvelet transform.

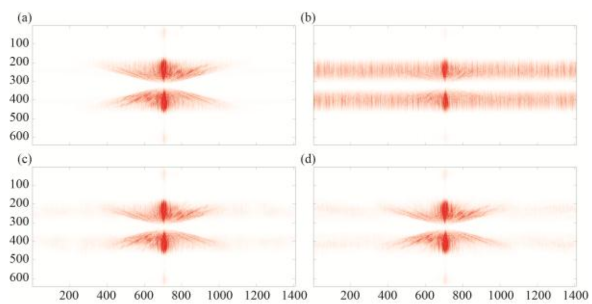

Figure 7: Fourier domain spectra of data in Figure 5. (a) Original data, (b) randomly sampled data, (c) recovered data using Dreamlet transform, (d) recovered data using Curvelet transform.

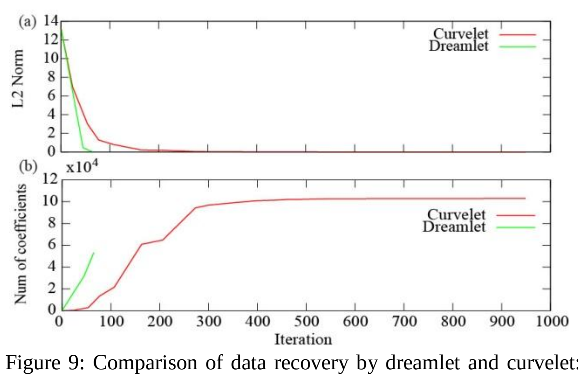

Figure 9: Comparison of data recovery by dreamlet and curvelet: l_2-norm of the residuals and number of coefficients as a function of iteration.

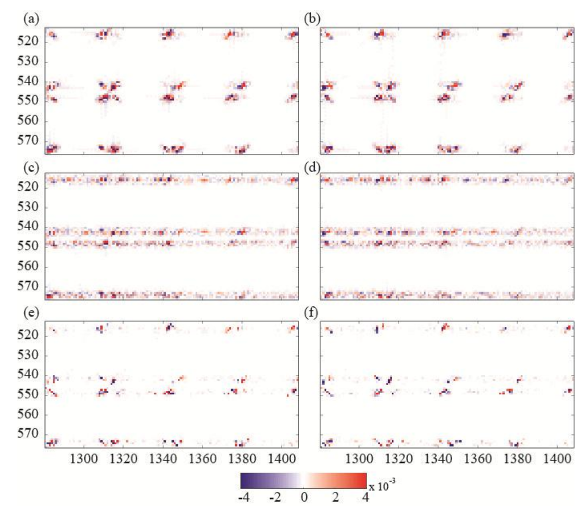

Figure 8: Coefficient matrix after Dreamlet decomposition (showing 8 blocks). (a), (b) real and imaginary part of original data, (c), (d) real and imaginary part of randomly sampled data, (e), (f) real and imaginary part of recovered data using Dreamlet transform.

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